

# On the Nature of the Evanescent Wave

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This is an unusual paper in that it does not address a particular research topic or present a novel experimental method or a new theoretical result. This paper addresses our basic understanding of the nature of the evanescent wave, the wave that is the basis of the entire field of Attenuated Total Reflection (ATR) spectroscopy. I recently had the opportunity to reexamine the foundations of ATR spectroscopy and was surprised to have had to change my own mental picture of the evanescent wave that I have built over the last 25 years. Over the years I have had numerous discussions with a large number of workers in the field as well as with my former mentor, and one of the originators and the principal developer of ATR spectroscopy, the late N.J. Harrick. Everything brought up in all these discussions was perfectly consistent with my old mental picture of the evanescent wave. Thus, I believe that the picture of the evanescent wave that I had is virtually universally held by workers in the field. This paper describes the new picture of the evanescent wave that emerged from said reexamination process.

Index Headings: Evanescent wave; ATR spectroscopy; Total internal reflection.

## INTRODUCTION

Attenuated total internal reflection (ATR) has become the most ubiquitous IR spectroscopic technique. The technique is based on the absorption of the evanescent wave that, in total internal reflection, forms on the low refractive index side along a totally reflecting interface. This wave is of great significance to ATR spectroscopy, and in this paper I hope to further clarify some characteristics of the evanescent wave. For the sake of specificity and consistent notation, in what follows, some of the well known elementary facts about electromagnetic theory are explicitly reiterated.

It is first demonstrated that electromagnetic theory demands that the electromagnetic wave is a transverse wave and that it propagates through a medium with speed  $c/n$  where  $c$  is the speed of light in vacuum and  $n$  is the refractive index of the medium. Then it is demonstrated that electromagnetic theory predicts both total internal reflection and the existence of the evanescent wave in total internal reflection.

What is puzzling is that the evanescent wave as provided by electromagnetic theory appears not to be a transverse wave and not to propagate through the medium by the speed demanded by the theory.

Furthermore, a conceptual difficulty arises with how the electromagnetic energy carried by the evanescent wave made it through the totally reflecting interface in the first place. To solve the mystery of where the electromagnetic energy in the evanescent wave comes from, the picture of the evanescent wave needs to be adjusted. This adjusted picture then removes all the apparent puzzles posed by the structure of the expression for the evanescent wave.

## STANDARD ELECTROMAGNETIC THEORY

We start with the concept of the electromagnetic wave. Maxwell equations in a homogeneous nonferromagnetic medium, characterized by the dielectric constant  $\epsilon$  and without sources, combine into the wave equation for both the electric and the magnetic field.<sup>1-4</sup> For the electric field of the electromagnetic wave we have

$$\left(\frac{\partial^2}{\partial \mathbf{x}^2} - \frac{\epsilon}{c^2} \frac{\partial^2}{\partial t^2}\right) \mathbf{E}(\mathbf{x}, t) = 0 \quad (1)$$

where  $\mathbf{E}(\mathbf{x}, t)$  is the amplitude of the electric field vector at the position  $\mathbf{x}$  and time  $t$  and  $c$  is the speed of light in vacuum. The solution to Eq. 1 is a plane wave:

$$\mathbf{E}(\mathbf{x}, t) = \mathbf{E}_0 e^{2\pi i n \mathbf{k} \cdot \mathbf{x} - i \omega t} \quad (2)$$

where  $n = \sqrt{\epsilon}$  is the refractive index of the medium.

The magnetic field of an electromagnetic wave can be expressed in terms of its associated electric field as

$$\mathbf{B} = \frac{2\pi c n}{\omega} \mathbf{k} \times \mathbf{E} \quad (3)$$

The wavevector  $\mathbf{k}$  points in the direction of wave propagation and has a magnitude (also known as wavenumber<sup>1</sup>)  $k$  given by

$$2\pi n k = \frac{\omega}{c} = \frac{2\pi n}{\lambda} \quad (4)$$

Here  $\omega$  is the angular frequency and  $\lambda$  is the wavelength of light in vacuum.

The phase in the exponential term in Eq. 2 represents the phase of oscillations of the electric field. For  $\mathbf{x}$  in the direction of  $\mathbf{k}$  a constant phase  $\psi$  in the exponent is achieved when

$$2\pi n k x - \omega t = \psi$$

This point of constant phase for the wave occurs when

$$x = \frac{\psi + \omega t}{2\pi n k} = \frac{\psi}{2\pi n k} + \frac{c}{n} t$$

In other words, the point of constant phase of the wave travels at the speed  $c/n$ . This is one of the results of the electromagnetic theory that I want to bring up.

As evident from Eq. 3 the two fields propagate together and oscillate in phase in every point in space. From Eq. 3 it is also evident that the two fields are mutually perpendicular, and both are perpendicular to the wave vector  $\mathbf{k}$ , which points in the direction of propagation. Since the electric and magnetic fields are both perpendicular to the direction of propagation, the electromagnetic wave is a transverse wave. This is another result of the electromagnetic theory that I want to bring up.

Maxwell equations for empty space free of sources and currents require that

$$\frac{\partial}{\partial t} \left[ \frac{1}{8\pi} (E^2 + B^2) \right] + \nabla \cdot \left[ \frac{c}{4\pi} (\mathbf{E} \times \mathbf{B}) \right] = 0 \quad (5)$$

The interpretation of expression 5 is that it describes conservation of energy associated with electromagnetic fields. The term

$$u = \frac{1}{8\pi} (E^2 + B^2) \quad (6)$$

is the energy density of the electromagnetic field. Since for electromagnetic waves the electric and magnetic fields have the same magnitudes, i.e.,  $B^2 = E^2$ , we have

$$u = \frac{1}{4\pi} E^2$$

The vector quantity

$$\mathbf{P} = \frac{c}{4\pi} (\mathbf{E} \times \mathbf{B}) = \frac{2\pi c^2}{\omega} u \mathbf{k} \quad (7)$$

is the so-called Poynting vector, which describes the flow of energy contained in the electromagnetic field. Note that the term  $2\pi c/\omega$  in Eq. 7 is just the inverse of the wavenumber. Thus the last expression in Eq. 7 is a vector whose magnitude is the product of the speed of light and energy density and that points in the direction of propagation. The Poynting vector thus describes the flow of energy density  $u$  at the speed of light, just as we would expect for energy carried by electromagnetic waves. Therefore electromagnetic energy flows with the same speed as the wave. This may sound as a tautology, but, as we will see, nothing can be taken for granted.

The presence of an interface (i.e., discontinuity in the medium) means that the solutions of Eq. 1 on one side of the interface do not extend to the other side. We have different solutions on the two sides. However, these solutions must come together at the interface. That requirement gives us the boundary conditions that then lead to not only Fresnel equations, but also the laws of reflection and refraction. At an interface between two media characterized by refractive indices  $n_1$  and  $n_2$  incident light refracts through and reflects from the interface. Thus we have incident, transmitted, and reflected waves. The incident and reflected waves are in the first (incident) medium, while the refracted (transmitted) wave is in the second medium. All three waves are plane waves described by Eq. 1. Each wave is of the form of Eq. 2 but has its own amplitude and wave vector:

$$\begin{aligned} \mathbf{E}_{in}(\mathbf{x}, t) &= \mathbf{E}_{0in} e^{2\pi i n_1 \mathbf{k}_in \cdot \mathbf{x} - i\omega t} \\ \mathbf{E}_r(\mathbf{x}, t) &= \mathbf{E}_{0r} e^{2\pi i n_1 \mathbf{k}_r \cdot \mathbf{x} - i\omega t} \\ \mathbf{E}_t(\mathbf{x}, t) &= \mathbf{E}_{0t} e^{2\pi i n_2 \mathbf{k}_t \cdot \mathbf{x} - i\omega t} \end{aligned} \quad (8)$$

The three wave vectors  $\mathbf{k}_{in}$ ,  $\mathbf{k}_r$ ,  $\mathbf{k}_t$  of the incident, reflected, and transmitted waves, respectively, lie in the same plane (plane of incidence), and their directions are determined by the laws of reflection and refraction. The field amplitudes  $\mathbf{E}_{0in}$ ,  $\mathbf{E}_{0r}$ ,  $\mathbf{E}_{0t}$  are the amplitudes of oscillations of the electric fields of incident, reflected, and transmitted waves, respectively. We

assume that the wave vector and the electric field amplitude of the incident wave are known. That means that we know the frequency and the direction of propagation of the incident wave. The theory then provides the wave vectors and the electric field amplitudes of both the reflected and transmitted waves.

The reflected and transmitted amplitudes are related to the incident amplitude through Fresnel equations. The ratios  $r$  and  $t$  are the Fresnel reflection and transmission amplitude coefficients respectively defined as

$$r = \frac{E_r}{E_{in}} \quad (9)$$

$$t = \frac{E_t}{E_{in}}$$

For  $s$ -polarized incident light,<sup>1-4</sup>

$$\begin{aligned} r^s &= \frac{N_1 \cos \theta - \sqrt{n_2^2 - n_1^2 \sin^2 \theta}}{N_1 \cos \theta + \sqrt{n_2^2 - n_1^2 \sin^2 \theta}} \\ t^s &= \frac{2n_1 \cos \theta}{n_1 \cos \theta + \sqrt{n_2^2 - n_1^2 \sin^2 \theta}} \end{aligned} \quad (10)$$

and similarly for  $p$ -polarized incident light,

$$\begin{aligned} r^p &= \frac{N_2^2 \cos \theta - n_1 \sqrt{n_2^2 - n_1^2 \sin^2 \theta}}{N_2^2 \cos \theta + n_1 \sqrt{n_2^2 - n_1^2 \sin^2 \theta}} \\ t^p &= \frac{2n_1 n_2 \cos \theta}{n_2^2 \cos \theta + n_1 \sqrt{n_2^2 - n_1^2 \sin^2 \theta}} \end{aligned} \quad (11)$$

For the reflected wave the cross section of the incident beam stays the same. But for the refracted wave it changes as<sup>3</sup>

$$\frac{C_t}{C_{in}} = \frac{\cos \varphi}{\cos \theta} \quad (12)$$

where  $C_t$  and  $C_{in}$  are the areas of the cross sections of transmitted and incident beams, respectively, and angles  $\theta$  and  $\varphi$  are the angle of incidence and refraction, respectively. Taking Eq. 12 into account the reflectance and transmittance of the interface are

$$R = \frac{P_r}{P_{in}} = \frac{E_r^2}{E_{in}^2} = |r|^2 \quad (13)$$

$$T = \frac{P_t}{P_{in}} = \frac{E_t^2 n_2 C_2}{E_{in}^2 n_1 C_1} = |t|^2 \frac{n_2 \cos \varphi}{n_1 \cos \theta} \quad (14)$$

In Eqs. 13 and 14,  $P_{in}$ ,  $P_r$ ,  $P_t$ , are the power densities carried by incident, reflected, and transmitted beams, respectively. By using Eqs. 10 and 11 in Eqs. 13 and 14, it can be explicitly shown that

$$T + R = 1$$

for either polarization. This, of course, must be so for the electromagnetic theory to comply with the law of conservation of energy.

### EVANESCENT WAVE

For internal reflection, light is incident from a material of higher refractive index  $n_1$  onto an interface with a material of lower refractive index  $n_2$ . The angle of refraction, given by Snell's law, is

$$\cos \theta = \sqrt{n_2^2 - n_1^2 \sin^2 \theta}. \quad (15)$$

The angle of refraction exists only for angles of incidence  $\theta$  smaller than the critical angle  $\theta_c$  given by

$$\sin \theta_c = \frac{n_2}{n_1} \quad (16)$$

For angles of incidence above the critical angle the term under the square root is negative, so Eq. 15 leads to an imaginary angle of refraction:

$$\cos \varphi = i\sqrt{n_1^2 \sin^2 \theta - n_2^2} \quad (17)$$

For angles of incidence below the critical, internal reflection is subcritical internal reflection, and above the critical angle, it is supercritical internal reflection.

For subcritical internal reflection the waves on the two sides of the interface are incident, reflected, and transmitted (Eq. 8), just as for external reflection. However, when the angle of incidence exceeds the critical angle, only incident and reflected waves remain. The transmitted wave is transformed into a so-called evanescent wave. To see how this comes about we select the coordinate system where the reflecting interface is the  $xy$  plane so that the  $z$  axis is normal to the interface, and we choose the plane of incidence to be the  $xz$  plane. The three wave vectors then have only  $x$  and  $z$  components. For the transmitted wave the scalar product in the exponent of its propagation factor can be evaluated as follows:

$$in_2k(x \sin \varphi + z \cos \varphi) = ikx n_1 \sin \theta - kz \sqrt{n_1^2 \sin^2 \theta - n_2^2} \quad (18)$$

This leads to

$$E_t(x, t) = E_0 e^{-i\omega t} e^{2\pi i n_1 k x \sin \theta} e^{-2\pi k z \sqrt{n_1^2 \sin^2 \theta - n_2^2}} \quad (19)$$

The first exponential term on the right-hand side of Eq. 19 is the standard oscillatory term that expresses the time dependence of the wave. The second term is just a standard term describing the propagation of the wave along the interface. These two terms look exactly the same as in the case of subcritical internal reflection. The last term describes the exponential decay with distance from the interface of the amplitude of the wave propagating along the interface. The wave is thus confined to the interface. The amplitude of the wave is largest at the interface and decreases exponentially with distance into the rarer medium. This surface wave, called the evanescent wave, is a remnant of the transmitted wave that lingers in the rarer medium at supercritical incidence.

It can be seen from Eq. 18 that the evanescent wave

propagates parallel to the interface with velocity  $c'$  given by

$$c' = \frac{c}{n_1 \sin \theta} \quad (20)$$

Since light propagates in medium 2 along the interface, it appears that the electromagnetic wave and electromagnetic energy propagate with the same speed, but that this speed is unrelated to the medium (characterized by  $n_2$ ) in which it propagates. Moreover, the speed of propagation can be fine tuned by changing the angle of incidence. This is in a blatant violation of what we found before, based on standard electromagnetic theory, about the speed of propagation of electromagnetic waves and electromagnetic energy.

Using the Fresnel equations 10 and 11 it is easy to express the electric field of the transmitted wave:<sup>1-4</sup>

$$E_t = t_{12} E_0 \quad (21)$$

For the  $s$ -polarized incident beam in the coordinate system as defined above this gives the electric field components as follows:

$$E_x = 0, E_y = \frac{2n_1 \cos \theta}{n_1 \cos \theta + \sqrt{n_2^2 - n_1^2 \sin^2 \theta}} E_0, E_z = 0 \quad (22)$$

and for the  $p$ -polarized incident light:

$$\begin{aligned} E_x &= \frac{2n_1 \cos \theta \sqrt{n_2^2 - n_1^2 \sin^2 \theta}}{n_2^2 \cos \theta + n_1 \sqrt{n_2^2 - n_1^2 \sin^2 \theta}} E_0, E_y = 0, E_z \\ &= -\frac{2n_1^2 \sin \theta \cos \theta}{n_2^2 \cos \theta + n_1 \sqrt{n_2^2 - n_1^2 \sin^2 \theta}} E_0 \end{aligned} \quad (23)$$

Thus, there is a component of the electric field of the  $p$ -polarized evanescent wave in the direction of propagation ( $x$  axis), a no-no for transverse waves.

From Eq. 23 it follows that for the  $p$ -polarized beam

$$E_x = -\frac{\sqrt{n_2^2 - n_1^2 \sin^2 \theta}}{n_1 \sin \theta} E_z \quad (24)$$

In the supercritical regime the square root in Eq. 24 is purely imaginary implying ( $i \equiv e^{i\pi/2}$ ) that  $E_x$  is oscillating  $90^\circ$  ( $\pi/2$ ) out of phase with  $E_z$ . When  $E_z$  is at maximum,  $E_x$  is at zero, and vice versa. If we imagine the oscillations of the electric field play out in time, the electric field vector rotates in the plane of incidence. The tip of the electric field vector describes an ellipse.<sup>3</sup> The evanescent wave propagates along the interface oscillating between a purely transverse wave ( $E_x = 0$ ) and a purely longitudinal ( $E_z = 0$ ) wave.

How can this be when we explicitly demonstrated that the electromagnetic wave must be transverse? It cannot be longitudinal, not even momentarily. Why is it that the same electromagnetic theory that earlier demanded strict transversality of the electromagnetic wave is now leading us to conclude that the  $p$ -polarized evanescent wave violates the transverse nature of electromagnetic waves? And how can it be that this unusual wave travels with a speed not related to the material in which it propagates? The results that we just

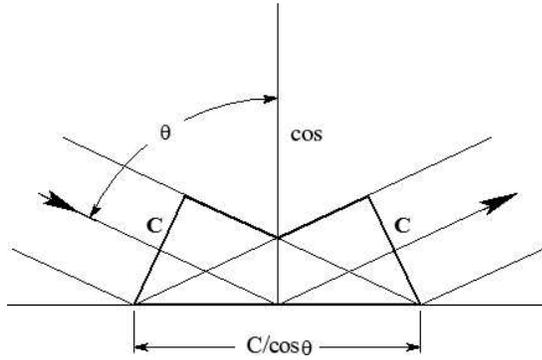


FIG. 1. Geometry of energy flow for total internal reflection.

derived from the electromagnetic theory clearly contradict the general results derived earlier from the same theory. Is the electromagnetic theory internally inconsistent?

### FLOW OF ELECTROMAGNETIC ENERGY THROUGH TOTALLY REFLECTING INTERFACE

From the Fresnel formulae 10 and 11 for nonabsorbing media 1 and 2 it follows that, for supercritical internal reflection, the reflectance amplitude coefficient is of the form<sup>3</sup>

$$r = \frac{X - iY}{X + iY} \quad (25)$$

Thus, since reflectance is the absolute value squared of the reflection amplitude coefficient, the reflectance is total, i.e.,  $R = |r|^2 = 1$ . This is true for both polarizations.

Therefore, for supercritical incidence, the amplitude reflection coefficients take a particularly simple form:

$$r = e^{i\alpha} \quad (26)$$

It is obvious from Eq. 9 that  $\alpha$  is the phase shift between the incoming and the reflected beam. Therefore, the totally reflected and the incident beam do not oscillate in phase. However, the intensity of a totally reflected beam is the same as the intensity of the incident beam. If all the electromagnetic energy is totally reflected at the interface, how is it then possible that there is an evanescent wave behind the interface? How did the electromagnetic energy make it through this seemingly impenetrable barrier?

On the other hand, we know that Fresnel equations connect fields on opposite sides of the interface, so Fresnel equations demand that, if there is an electromagnetic field on the incident side of the interface, there must be an electromagnetic field (and thus electromagnetic energy as well) behind the interface. Again we run into an apparent contradiction.

Imagine a beam of a laser pointer totally reflecting at an interface. The beam illuminates a small spot on the interface. Equation 19 apparently states that there is an evanescent wave in the illuminated area on the other side of the interface propagating along the interface with the speed (Eq. 20). What happens to this evanescent wave when it reaches the edge of the illuminated area? It cannot just continue propagating since it would carry away with it some of the electromagnetic energy. This energy would then be missing from the reflected beam and the total reflection would not be total. This is yet

another seemingly contradictory result derived from the standard electromagnetic theory.

### RESULTS AND CONCLUSION

Let us start by explicitly expressing the flows of electromagnetic energy carried by the three beams that participate in reflection. According to Eq. 7, with the help of Eqs. 6 and 9, we can write

$$\begin{aligned} P_{in} &= \frac{cn_1}{4\pi} E_o^2 \\ P_r &= \frac{cn_1}{4\pi} (r^p)^2 E_o^2 \\ P_t &= \frac{cn_2}{4\pi} (t^p)^2 E_o^2 \end{aligned} \quad (27)$$

Note the shape of the volume outlined with a heavier line in Fig. 1. The incident beam enters through the left side, whereas the reflected beam exits through the right side. The base of the shape is in the interface, and any flow of energy through the interface to and from the evanescent wave is given by the component of  $P_t$  perpendicular to the interface  $\{(P_t)_z = P_t \cos \varphi\}$ . The cross-sectional areas of the incident and reflected beam are indicated by letters  $C$  and the area of the interface illuminated by the incident beam by  $C/\cos \theta$ .

In order for the electromagnetic powers flowing in the incident, reflected, and transmitted beams to balance, we must have

$$P_{in} - P_r = P_t \frac{\cos \varphi}{\cos \theta} \quad (28)$$

Using Eqs. 27 and 11 it is straightforward to verify that Eq. 28 is indeed satisfied. Surprisingly, we did not even have to sort out how to deal with the complex number values of the variables in Eq. 28. Not surprisingly, energy is conserved during total internal reflection. However, notice that in Eq. 28 we had to explicitly take into account that energy flows through the totally internally reflecting surface. Thus, not only does electromagnetic energy flow through the totally reflecting interface, but we can see that the evanescent wave actually serves as an energy "overflow tank" to allow for momentary energy flow imbalances between the incoming and reflected beams. An electric capacitor is an apt analog. In hindsight it is obvious that some overflow handling mechanism must exist if energy is to be conserved at every moment during total internal reflection. Because incoming and reflected waves are shifted in phase, there must be a mechanism in place that allows the momentary energy flow imbalance between the two beams to be taken away and supplied back as needed.

This vigorous energy flow through a totally internally reflecting interface seems to be contradictory to our earlier interpretation of the expression for the evanescent wave (Eq. 19). Then we concluded that energy flows parallel to the interface, and now we find that the energy flow parallel to the interface does not even enter into the energy balance (Eq. 28). How can this be? Is our interpretation of energy carried by the evanescent wave in some way wrong? And what about our finding that the evanescent wave is not a transverse wave and also that it does not travel with the proper speed for

electromagnetic waves in the medium? Let us now reanalyze all these findings.

First, let us reevaluate the issue of the transversality of the evanescent wave. Only the  $p$ -polarized wave causes problems, so we can calculate the scalar product of the electric field vector (Eq. 23) and the wavevector  $\mathbf{k}_t = k(n_1 \sin \theta, 0, [n_2^2 - n_1^2 \sin^2 \theta]^{1/2})$  of a  $p$ -polarized evanescent wave. Surprisingly (or not), we find that it is exactly zero. Now, by definition, two vectors are perpendicular if their scalar product is zero. Our intuition was misled by complex numbers appearing as magnitudes of the components of these vectors. However, if everything is taken into account, it all works out, and the evanescent wave reemerges as a transverse wave. It is thus demonstrated that nontransversality of the  $p$ -polarized evanescent wave is not born out once it is formally tested. So, although the  $p$ -polarized evanescent wave looks like it is violating the transversality requirement, it is not, and it is intellectually satisfying that this can be explicitly demonstrated.

But what kind of a wave is then the evanescent wave? It cannot be a propagating wave because its speed is wrong for a propagating wave. Incidentally, the speed of propagation of the evanescent wave is not just some abstract mathematical entity. The ability to fine tune the speed of the evanescent wave is used as a tool to excite so-called surface plasma waves (plasmons). The plasma waves are collective excitations of free electron charge density that are, like the evanescent wave, confined to the surface of metal. These waves propagate with a speed that is a function of their frequency. For the electric field of the evanescent wave to excite plasma waves, the two not only have to have the same frequency, but also have to travel with the same speed. Otherwise they would quickly run out of resonance. So how are all these facts mutually reconcilable? The answer is that the propagation of the evanescent wave parallel to the interface is a chimera. Nothing actually propagates, but what mimics the propagation is that electromagnetic energy bobbing back and forth through the interface is phase coordinated. The phase of these oscillations travels along the interface with the speed  $c'$  (Eq. 20). The picture of this is not unlike a wave on the surface of water. The water molecules move only up and down, but their phase is coordinated so we perceive a wave traveling on the surface. At the edge of the illuminated area the sustaining action of the incoming wave stops, and, thus, the evanescent wave stops. But nothing physical stops here because nothing physical actually ever moved along the interface. Only the phase of coordinated oscillations of energy through the interface stops propagating. As referenced in *Internal Reflection and ATR Spectroscopy*, a screw thread analogy is illustrative.<sup>3</sup> As a screw turns around its axis, the thread on top (or anywhere else) of the screw appears as if traveling along the screw axis. However, nothing really moves along the axis. Any point on the screw undergoes purely rotational motion around the screw axis. At the end of the screw, the apparent motion of the thread just suddenly stops as does the evanescent wave at the edge of the illuminated spot discussed above. It is interesting that a textbook<sup>4</sup> indeed recognizes the problem of how the energy from an incident wave gets through a totally reflecting interface and into an evanescent wave. These authors offered a solution that invokes a finite duration of an actual electromagnetic signal, which then necessitates the transitional interval from “no wave” to “wave” during which the expressions that we

derived are not valid and the transfer of energy through totally reflecting interface is thus not forbidden. However, as shown here, the problem is a chimera that goes away once we realize that there is a vigorous back and forth flow of energy through the interface at all times. There is, indeed, no need for any explanation including the one offered by said textbook.

Finally, let us consider in some detail the speed of the evanescent wave. The speed of a wave is by definition the ratio of the wavelength  $\lambda$  (distance traveled) and the period of oscillations  $\tau$  (time it takes to travel the distance of one wavelength):

$$c = \frac{\lambda}{\tau} = \lambda \nu \quad (29)$$

The second expression in Eq. 29 follows since the inverse of the period of oscillations is, by definition, the frequency  $\nu$  of oscillations. The angular frequency is  $\omega = 2\pi\nu$ . The inverse of the wavelength is the wavenumber  $k$ . Thus,

$$\omega = 2\pi ck \quad (30)$$

For the incident wave  $k_{in} = n_1 k$ , so the speed of the incident wave is  $c/n_1$ . For the evanescent wave the situation is again a bit confusing since the wave vector (as indicated in Eqs. 18 and 19) has the imaginary  $z$  component (perpendicular to the interface):

$$\begin{aligned} \mathbf{k}_t &= k(n_2 \sin \varphi, 0, i\sqrt{n_2^2 - n_1^2 \sin^2 \theta}) \\ &= k(n_1 \sin \theta, 0, i\sqrt{n_2^2 - n_1^2 \sin^2 \theta}) \end{aligned}$$

where we used Snell's law to get to the second expression in the above equation. Now, taking the scalar product of the wave vector with itself yields

$$k_t^2 = k^2(n_1^2 \sin^2 \theta - n_1^2 \sin^2 \theta + n_2^2) = (n_2 k)^2 \quad (31)$$

which, interestingly, gives for the speed of the evanescent wave  $c/n_2$ , the correct speed for light in medium 2, not Eq. 20. Thus, there is formally no discrepancy between the properties of the evanescent wave and other propagating electromagnetic waves.

In conclusion we can say that, in the end, the electromagnetic theory is vindicated. It does not contradict itself in its description of the evanescent wave. The impression that it does stems from misconceptions about its nature, i.e., the inaccurate mental picture of what the evanescent wave is. When everything is worked out consistently, the evanescent wave emerges as a transverse wave whose nominal speed of propagation is correct, although it actually does not propagate through the medium in which it exists. It also emerges that the flow of electromagnetic energy through a totally internally reflecting interface is not only vigorous but also essential to enabling the conservation of energy at any instant during total internal reflection. The evanescent wave is not a propagating wave in the usual sense, but it does formally retain all the standard characteristics of a propagating electromagnetic wave. These characteristics are inherited from the transmitted wave and survive the transition from the subcritical to the supercritical regime of internal reflection. The confusion comes about from the fact that, in that transition, some of the quantities that describe and define the electromagnetic wave

flip from having real to having imaginary magnitudes, which induces confusion into the interpretation of what these quantities now mean. A standard textbook<sup>1</sup> describes the evanescent wave as propagating along the interface. The leading optics textbook<sup>4</sup> simply accepts the apparent non-transversality of the evanescent wave, labels the evanescent wave as inhomogeneous, gives no justification for exempting inhomogeneous waves from the requirements of electromagnetic theory, and proceeds on to other issues. As demonstrated in this paper, the evanescent wave is not a propagating wave despite its appearing as if it is propagating along the interface. If it were a propagating wave, it would be subject to scattering in those cases where medium 2 is powder, and, therefore, the supercritical internal reflection of a nonabsorbing powdered sample would not be total. It is. Finally, when formally calculated, the “speed” of propagation of the evanescent wave was shown to be  $c/n_2$ , which is as required by electromagnetic theory. The impression left by the textbooks is that the evanescent wave is indeed unusual, even a strange kind of electromagnetic wave, but they do not go into sufficient detail

to fully illuminate its nature. Also, since total internal reflection is an old and well-known phenomenon, it is not seen as a promising area of research, so the nature of the evanescent wave does not receive much attention. No new and unknown phenomena are expected in the familiar territory of electromagnetic theory. Although no new phenomena were uncovered in this reanalysis, we see that there was indeed something left to be learned about the evanescent wave.

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